PREPARATION FOR RESEARCH WITH PROFESSOR GIBBONS

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1. Initial Tasks

(1) Complete Modern Algebra (Math 325W).
(2) Complete the following tutorials on the Home tab of http://web.macaulay2.com:
    (a) Welcome Tutorial
    (b) Basic Introduction to Macaulay2
    (c) Mathematicians' Introduction to Macaulay2 (first two parts)
(3) Read parts of this: https://arxiv.org/abs/1606.01867:
    (a) Read through §1 for a background on the types of problems Professor Gibbons works on.
    (b) Skim §3 and subsection §9.3 for the particular type of results that Professor Gibbons is currently interested in.
(4) There are a few papers that drive Professor Gibbons's current research in this area. You shouldn't read them in their entirety, but you may wish to get a "flavor" of the results in these areas by skimming through them.
    (a) https://arxiv.org/abs/1106.0381
    (b) https://digitalcommons.hamilton.edu/articles/58/
    (c) https://digitalcommons.hamilton.edu/articles/60/
(5) Look at these papers for examples of research conducted by Professor Gibbons and students:
    (a) https://digitalcommons.hamilton.edu/articles/269/
    (b) https://digitalcommons.hamilton.edu/articles/275/
(6) Complete the homework problems on the next page. Where helpful, references from the above papers are included to give you some focused reading with the papers above.

Date: January 18, 2019.

2. Homework Problems

Feel free to visit office hours to discuss these problems.

(1) **Using Macaulay2 for Betti diagram calculations** (Required)
When you accomplish all the tasks below, save your input and output buffers. You should make sure that you clean up your code so that you have the cleanest possible output buffer. That is, you will probably run through your code a few times, tweak it, and then rerun it so it doesn't have any errors in the output buffer before you save your buffers. To restart your Macaulay2 session, use the command `restart`.

Go to web.macaulay2.com and encode the following matrix:

$$\begin{bmatrix} 3 & 3 & 0 \\ 0 & 3 & 3 \end{bmatrix}$$

When you create this matrix, name it A.

Next, load the package `BoijSoederberg` with the command:
`loadPackage "BoijSoederberg"`
and run the command:
`B = mat2betti(A)`

Use the `BoijSoederberg` package to decompose the Betti diagram B into pure diagrams with fractional entries arising from the Herzog-Kühl equations. For help on this, see the documentation for the commands `decomposeBetti` and `TableEntries` (you'll need to make sure you have the `BoijSoederberg` package loaded to use the Macaulay2 `help` or `viewHelp` functions with these commands).

Make a ring R with coefficients in the field $\mathbb{Z}/8821$ and variables x, y. Create the ideal I generated by x^2, xy, y^2 and use Macaulay2 to calculate the betti diagram of R/I. Name this Betti diagram B_1. For help on this, view the documentation for the command `resolution` and see what happens when you ask for the resolution of an ideal.

Create a module M with the code `M = cokernel matrix{{x,-y,0},{0,x,-y}}` and find the Betti table of M; name the Betti table B_2.

Check to see if $B_1 + B_2 = B$.

(2) **Computing syzygies** (Optional)
By hand, find the free resolution of the modules

$$M_1 = k[x,y]/(x^2, xy, y^2) \text{ and } M_2 = \operatorname{coker} \begin{bmatrix} x & -y & 0 \\ 0 & x & -y \end{bmatrix}.$$

Describe (in one word!) how the matrices in the free resolutions of these modules are related.

(3) **Combinatorial Properties of Complete Intersections**
(Required; see Gibbons-Jeffries-Mayes-Raicu-Stone-White) By hand, calculate following numerical invariants of the complete intersection $k[x,y,z]/(x^2, y^3, z^5)$:
   (a) The Hilbert function
   (b) The Betti diagram

(4) **Betti diagrams with infinite support** (Required; see Berkesch-Burke-Erman-Gibbons)
   (a) Which of the following Betti diagrams are extremal in the cone of Betti diagrams over the ring $k[x,y]/(x^2)$? (Circle all that apply)
      (i) $\beta(R) = \begin{pmatrix} 1 & - & - & - \cdots \end{pmatrix}$
      (ii) $\beta(R/(x)) = \begin{pmatrix} 1 & 1 & 1 & 1 & \cdots \end{pmatrix}$
      (iii) $\beta(R/(y)) = \begin{pmatrix} 1 & - & - & - & \cdots \end{pmatrix}$
      (iv) $\beta(R/(x,y)) = \begin{pmatrix} 1 & 2 & 2 & 2 & \cdots \end{pmatrix}$
   (b) Give another example of an extremal Betti diagram over $k[x,y]/(x^2)$
   (c) Give another example of a non-extremal Betti diagram over $k[x,y]/(x^2)$